4 **(a)** Given that $\log_{\sqrt{2}} x = m$ and $\log_4 y = n$, express $\dfrac{\sqrt{y}}{x^6}$ in terms of m and n. [4]

(b) Solve the equation $\log_2(3x-2) = \log_4\left(x^2+1\right) + \dfrac{2}{\log_{\sqrt{2}} 2}$. [6]

4 **(a)** Given that $\log_{\sqrt{2}} x = m$ and $\log_4 y = n$, express $\dfrac{\sqrt{y}}{x^6}$ in terms of m and n. [4]

$\log_{\sqrt{2}} x = m$

$x = \sqrt{2}^{m}$

$\log_4 y = n$

$y = 4^{n}$

$$\frac{\sqrt{y}}{x^6} = \frac{\sqrt{4^n}}{(\sqrt{2}^{m})^6}$$

$$= \frac{2^n}{2^{3m}}$$

$$= 2^{n-3m}$$

Ans

(b) Solve the equation $\log_2(3x-2) = \log_4\left(x^2+1\right) + \dfrac{2}{\log_{\sqrt{2}} 2}$. [6]

$$\log_2(3x-2) = \frac{\log_2(x^2+1)}{\log_2 4} + \frac{2}{\left(\frac{\log_2 2}{\log_2\sqrt{2}}\right)}$$ (change of base)

$$\log_2(3x-2) = \frac{\log_2(x^2+1)}{2} + \frac{2\log_2\sqrt{2}}{1}$$

$$\log_2(3x-2) = \frac{\log_2(x^2+1)}{2} + \log_2\left(2^{\frac{1}{2}}\right)^2$$ (denominator)

$$2\log_2(3x-2) = \log_2(x^2+1) + \log_2(2)^2$$

$$\log_2(3x-2)^2 = \log_2 4(x^2+1)$$

$\therefore$ By comparison, $(3x-2)^2 = 4(x^2+1)$

$9x^2-12x+4 = 4x^2+4$

$5x^2-12x = 0$

$x=0$ or $x = \frac{12}{5}$ // Ans

(rejected)

reconstitute into original equation,
$x=0$ will give $\log_2(3x-2)$ = undefined.

Anderson S4 A Math 2020 P2 Q4