4 **(i)** By completing the square, express $x^2 - x + 1$ in the form $(x-p)^2 + q$ where p and q are constants. [2]

(ii) Show that the curve $y = x^2 - 2px + p - 1$ will always cut the x-axis at two distinct points for all real values of p. [3]

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$$x^2 - x + 1 = x^2 - x + \left(-\tfrac{1}{2}\right)^2 - \left(-\tfrac{1}{2}\right)^2 + 1$$
$$= \left(x - \tfrac{1}{2}\right)^2 + \tfrac{3}{4}$$

(ii) Show that the curve $y = x^2 - 2px + p - 1$ will always cut the x-axis at two distinct points for all real values of p. [3]

$$\text{Discriminant} = (-2p)^2 - 4(1)(p-1)$$
$$= 4p^2 - 4p + 4$$
$$= 4(p^2 - p + 1)$$
$$= 4\left[\left(p - \tfrac{1}{2}\right)^2 + \tfrac{3}{4}\right]$$
$$= 4\left(p - \tfrac{1}{2}\right)^2 + 3$$

Since $\left(p - \frac{1}{2}\right)^2$ is never negative for all real values of p, thus the discriminant $4\left(p - \frac{1}{2}\right)^2 + 3$ will always be positive. Hence the curve will always cut the x-axis at two distinct points.