8 Given that $\dfrac{4x^2-x-3}{2x^2-x}=a+\dfrac{bx+c}{2x^2-x}$,

(i) find the value of each of the integers a, b and c. [2]

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(ii) Using the values of b and c obtained in part **(i)**, express $\dfrac{bx+c}{2x^2-x}$ in partial fractions. [4]

Hence, using parts **(i)** and **(ii)**, find

(iii) $\int \frac{8x^2 - 2x - 6}{2x^2 - x}\, \mathrm{d}x.$ [4]

8 Given that $\dfrac{4x^2-x-3}{2x^2-x} = a + \dfrac{bx+c}{2x^2-x}$,

(i) find the value of each of the integers a, b and c. [2]

$$\frac{4x^2-x-3}{2x^2-x} = \frac{2(2x^2-x)+x-3}{2x^2-x}$$

$$= 2 + \frac{x-3}{2x^2-x}$$

$$\therefore a = 2,\ b = 1 \text{ and } c = -3$$ ans

(ii) Using the values of b and c obtained in part **(i)**, express $\dfrac{bx+c}{2x^2-x}$ in partial fractions. [4]

$$\frac{x-3}{2x^2-x} = \frac{x-3}{x(2x-1)}$$

$$= \frac{A}{x} + \frac{B}{2x-1}$$

$$= \frac{A(2x-1)+Bx}{x(2x-1)}$$

$$x-3 = A(2x-1)+Bx$$

Let $x = 0$,

$$0-3 = A(-1)$$

$$A = 3$$

Let $x = 0.5$,

$$-2.5 = 0.5B$$

$$B = -5$$

$$\therefore \frac{x-3}{2x^2-x} = \frac{3}{x} - \frac{5}{2x-1}$$

Hence, using parts **(i)** and **(ii)**, find

(iii) $\int \frac{8x^2-2x-6}{2x^2-x}\, dx.$ [4]

$$\int \frac{8x^2-2x^2-6}{2x^2-x}\, dx$$

$$= 2\int \frac{4x^2-x-3}{2x^2-x}\, dx$$

$$= 2\int 2+\frac{3}{x}-\frac{5}{2x-1}\, dx$$

$$= 2\left(2x+3\ln x-\frac{5}{2}\ln(2x-1)\right)+c$$

$$= 4x+6\ln x-5\ln(2x-1)+c$$

where c is a constant