8 (a) Given that $y = 3xe^{-2x}$, find

(i) $\frac{dy}{dx}$,

(ii) the value of p if $p = e^{2x}\left(\frac{d^2y}{dx^2} + \frac{dy}{dx} - 2y\right)$. [4]

8 (a) Given that $y = 3xe^{-2x}$, find

(i) $\frac{dy}{dx}$, [3]

$y = 3xe^{-2x}$

$\frac{dy}{dx} = 3x \frac{d}{dx}(e^{-2x}) + e^{-2x} \frac{d}{dx}(3x)$ Product Rule

$= 3x \cdot (-2) \cdot e^{-2x} + e^{-2x} \cdot 3$

$\frac{dy}{dx} = 3e^{-2x}[-2x + 1]$ // Ans.

(ii) the value of p if $p = e^{2x}\left(\frac{d^2y}{dx^2} + \frac{dy}{dx} - 2y\right)$. [4]

$y = 3xe^{-2x}$

$\frac{dy}{dx} = 3e^{-2x}[-2x + 1]$

$\frac{d^2y}{dx^2} = 3e^{-2x} \cdot (-2) + [-2x+1] \cdot (-6e^{-2x})$

$= -6e^{-2x}[1 + (-2x+1)]$

$= -6e^{-2x}[-2x + 2]$

$= -12e^{-2x}[-x + 1]$

$= 12e^{-2x}[x - 1]$

∴ By substitution, $p = e^{2x}\left(12e^{-2x}[x-1] + 3e^{-2x}[-2x+1] - 2(3xe^{-2x})\right)$

$= 12(x-1) + 3(-2x+1) - 2(3x)$

$= 12x - 12 - 6x + 3 - 6x$

$= -9$

Reason:
$e^{2x} \cdot e^{-2x} = 1$