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25 days after first being observed, the mass has dropped to 10.3 g.

(i) Find the initial mass of the radioactive substance. [1]

(ii) Find the number of days it takes after first being observed for the mass to become half its original value. [5]

(iii) Find the rate at which the mass is decreasing when $t = 40$ days. [2]

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25 days after first being observed, the mass has dropped to 10.3 g.

(i) Find the initial mass of the radioactive substance. [1]

$t = 0$, $m = 38g$? $m = 38e^{kt}$ $t=0$

$m = 38e^{k(0)}$

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(ii) Find the number of days it takes after first being observed for the mass to become half its original value. [5]

Given $t=25$, $m=10.3$

$38e^{k(25)} = 10.3$

$\therefore k = -0.052217$

For half mass, $m_{\frac{1}{2}} = \frac{38}{2} = 19$

$\therefore 38e^{-0.052217t} = 19$

$t = 13.274$ days

$= 13.3$ days (3 sf)

≈ 14 days (to go past half mass)

(iii) Find the rate at which the mass is decreasing when $t = 40$ days. [2]

$$\frac{dm}{dt} = 38ke^{kt}$$

$$= 38(-0.052217)e^{-0.052217 \times 40}$$

$$\frac{dm}{dt} = -0.246 \text{ (3sf)}$$

mass is decreasing at a rate of 0.246 g/s