4 **(a)** A triangle has a base of $\left(10+\sqrt{3}\right)$ cm and has an area of $\left(8+\sqrt{12}\right)$ cm^2. **Without using a calculator**, find the height of the triangle, in the form of $\left(p+q\sqrt{3}\right)$ cm, where p and q are rational numbers. [4]

(b) Given that $1+\sqrt{2}$ is a root of the equation $ax^2+bx+c=0$ where a, b and c are integers, show that $a+c=0$. [3]

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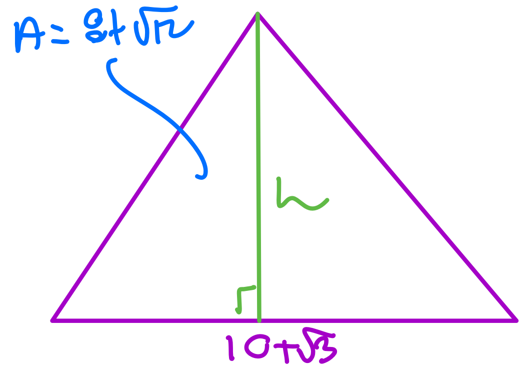

Let the height of the triangle be h

$$\frac{1}{2}(10+\sqrt{3})h = 8+\sqrt{12}$$

$$h = \frac{2(8+\sqrt{12})}{10+\sqrt{3}}$$

$$= \frac{16+2\sqrt{12}}{10+\sqrt{3}} \times \frac{10-\sqrt{3}}{10-\sqrt{3}}$$ (Rationalise)

$$= \frac{160-16\sqrt{3}+20\sqrt{12}-2\sqrt{36}}{100-3}$$

$$= \frac{160-16\sqrt{3}+40\sqrt{3}-12}{97}$$

$$= \frac{148+24\sqrt{3}}{97}$$

$$= \frac{148}{97}+\frac{24}{97}\sqrt{3}$$

(b) Given that $1+\sqrt{2}$ is a root of the equation $ax^2+bx+c=0$ where a, b and c are integers, show that $a+c=0$. [3]

For root, $x = 1+\sqrt{2}$ is substituted into ax^2+bx+c

$a(1+\sqrt{2})^2 + b(1+\sqrt{2}) + c = 0$

$a(1^2+2\sqrt{2}+2) + b(1+\sqrt{2}) + c = 0$

$a(3+2\sqrt{2}) + b(1+\sqrt{2}) + c = 0$

$(3a+b+c) + (2a+b)\sqrt{2} = 0$

$\Rightarrow 3a+b+c = 0 - (1)$ and $2a+(1)+(2)$

$a+c=0$

Hint:

$(3a+b+c) + (2a+b)\sqrt{2} = 0 \rightarrow 0 + 0\sqrt{2}$

(imagine $0 = 0 + 0\sqrt{2}$)

compare with LHS = RHS