$$\frac{2-\operatorname{cosec}^2 A}{\operatorname{cosec}^2 A + 2\cot A} = \frac{\sin A - \cos A}{\sin A + \cos A}$$

$$\text{LHS} = \frac{2-\operatorname{cosec}^2 A}{\operatorname{cosec}^2 A + 2\cot A}$$

$$= \frac{2-(1+\cot^2 A)}{1+\cot^2 A + 2\cot A}$$

$$= \frac{2-1-\cot^2 A}{(\cot A + 1)^2}$$

$$= \frac{1-\cot^2 A}{(\cot A+1)^2}$$

$$= \frac{(1+\cot A)(1-\cot A)}{(\cot A+1)^2}$$

$$= \frac{1-\cot A}{1+\cot A}$$

$$= \left(1 - \frac{\cos A}{\sin A}\right) \div \left(1 + \frac{\cos A}{\sin A}\right)$$

$$= \left(\frac{\sin A - \cos A}{\sin A}\right) \div \left(\frac{\sin A + \cos A}{\sin A}\right)$$

$$= \frac{\sin A - \cos A}{\sin A} \times \frac{\sin A}{\sin A + \cos A}$$

$$= \frac{\sin A - \cos A}{\sin A + \cos A} \quad \text{(proved)}$$